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A SHORT NOTE ON ASSOCIATED MATRICES

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Abstract: *In this paper we define Fibonacci Q matrix and discuss associated matrices with related properties.*

Key words: Fibonacci numbers, Lucas numbers, matrix methods

Introduction

Fibonacci sequence and Lucas sequence has many interesting properties. The Fibonacci Q matrix is given by

$$Q = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Satisfies the relation (see [1])

$$Q^2 = Q + I_2$$

where I_2 is the 2 x 2 unit matrix of order 2. This relation further implies that

$$Q^{n+2} = Q^{n+1} + Q^n , \qquad n \geq 0$$

where Q^0 means the unit matrix .

Definition 1.1:

Associated matrices: Given any two FLM's $\{A_n\}$ and $\{B_n\}$ of order 2, is said to be an associated matrix for the pair (A_n, B_n), if the following relation holds

$$R(A, B) \text{ x } A_n = B_n , \qquad n \geq 0$$

Example: If F(n) and L(n) are two matrices given by

$$F(n) = \begin{bmatrix} Fn+1 & Fn \\ Fn & Fn-1 \end{bmatrix} \quad \text{and}$$

$$L(n) = \begin{bmatrix} Ln+1 & Ln \\ Ln & Ln-1 \end{bmatrix} \quad \text{respectively and if}$$

R(F,L) = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ then it is know that from [3]

R(F,L) x F(n) = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ X $\begin{bmatrix} Fn+1 & Fn \\ Fn & Fn-1 \end{bmatrix}$ = $\begin{bmatrix} Ln+1 & Ln \\ Ln & Ln-1 \end{bmatrix}$ = L(n) ; $n \geq 0$

This shows that the matrix R(F,L) is an associated matrix for the pair (F(n),L(n)).

Preliminaries

If the matrix R(F,L) coincides with the matrix L(0). This happens because of the definition of matrix product and connective property.

$L_n = F_n + 2F_{n-1} = 2F_{n+1} - F_n$ (2.1)

If we assume

R(F,L) = $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ (2.2)

and determine the unknown numbers x,y,z,w using the relation

R(F,L) x F(n) = L(n) (2.3)

With n=0, we get

R(F,L) = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ = $\begin{bmatrix} L1 & L0 \\ L0 & L-1 \end{bmatrix}$ (2.4)

We also have same result for R (F,L)

R(F,L) = $\begin{bmatrix} L1 & L0 \\ L0 & L-1 \end{bmatrix}$ x $\begin{bmatrix} Fn+1 & Fn \\ Fn & Fn-1 \end{bmatrix}$ = $\begin{bmatrix} Ln+1 & Ln \\ Ln & Ln-1 \end{bmatrix}$ (2.5)

i.e.

L(0) x F(n) = L(n) (2.6)

similarly

L(1) x F(n) = L(n+1) (2.7)

L(2) x F(n) = L(n+2) (2.8)

From this we can generalize a conjecture,

L(m) x F(n) = L(m+n) where m is any non-negative integer.

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This result when expressed in the matrix form, we have

$$\begin{bmatrix} Lm+1 & Lm \\ Lm & Lm-1 \end{bmatrix} X \begin{bmatrix} Fn+1 & Fn \\ Fn & Fn-1 \end{bmatrix} = \begin{bmatrix} L\,m+n+1 & Lm+n \\ Lm+n & Lm+n-1 \end{bmatrix} \quad (2.9)$$

This identity can be proved by using the principle of strong induction on m,n.

Omitting the details and by using (2.9), we have theorem 3 as follows:

Theorem

A) $L_{m+1}.F_{n+1} + L_mF_n = L_{m+n+1}$

B) $L_{m+1}.F_n + L_m.F_{n-1} = L_{m+n}$

C) $L_m.F_{n+1} + L_{m-1}.F_n = L_{m+n}$

D)$L_m.F_n + L_{m-1}.F_{n-1} = L_{m+n-1}$

These four properties are related to each other and obtained from (2.9).

Proof:

B) is obtained by equating upper right of 2.9

C) we can find by equating lower left of 2.9 ,interchanging the role of m and n in B) it is also possible to get C).

Replacing n by n+1 in B), we get A).

Replacing m by m-1 in B) one can find D).

Conclusion

Using definition of associated matrices, properties related to Fibonacci and Lucas sequences is obtained.

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